Energy Management Under Policy and Technology Uncertainty

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APPENDIX

SMAA-TY relies upon the capacity of *Analytica* to facilitate Monte Carlo Analysis (MCA) for customized Beta probability distributions, thereby reproducing the uniform distribution of stakeholder weights (under constraints) generated by the original SMAA algorithm. Analytica's default configuration generates pseudo-random values with Park & Miller's Minimal Standard algorithm with a Bays-Durham shuffle, samples with a pure Monte Carlo method, and provides a set of standard distribution functions including Normal, Triangular, and Beta. Additional options exist, but were not employed to generate with algorithms from "L'Ecuyer" and "Knuth", and to sample with Median Latin Hypercube and Random Latin Hypercube. To ensure the validity of the SMAA-TY approach, a number of simulations have been undertaken to test the relation of the generated weights to the Beta distributions specified.

SMAA-TY's Pseudo Markov-Chain: Unconstrained Weights

In the absence of any information with regard to stakeholder preferences, weights can be modeled as completely unconstrained. In this case, all mathematically feasible weight combinations must be tested, such that the range of each criterion is from very near zero to very nearly 100%. SMAA-TY generates each weight in a step-wise fashion in which the weight of the first criterion is sampled from a Beta distribution parameterized for the total number of criteria N, where $\alpha = 1$ and $\beta = N - 1$.

$$f(x;\alpha,\beta) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}\,du}$$

Successive criteria weights are generated from subsequent Beta distributions that are re-parameterized, given the assignment of the first weight (and the remaining unassigned weight). Each sampling proceeds with successively fewer criteria and a smaller weight space. For example, for $N = 4$, we call the first weight W4, which is sampled randomly from values between 0 and 100 with a Beta function of $\alpha = 1$, $\beta = 3$. (See Figure A1 below).

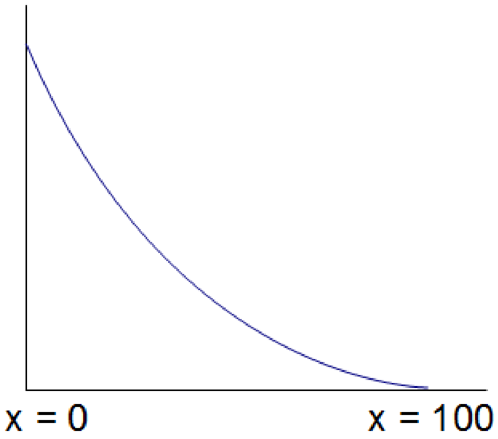

Figure A1. Beta distribution, α = 1, β = 3 for unconstrained assignment of first criterion weight in step-wise fashion, when the total number of criteria *N* = 4.

The second iteration will generate a value for one less number of criteria, and reduce the range of the feasible space by the weight already assigned to the first criterion. That involves generating a value between 0 and (100-W4) according to the Beta function $\alpha = 1$, $\beta = 2$. We will refer to this value as W3, which is sampled from the distribution represented in Figure A2.

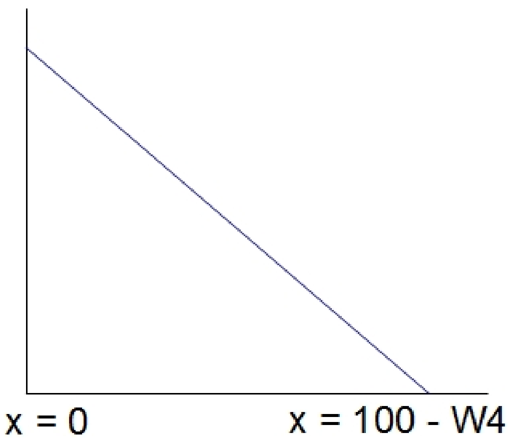

Figure A2. Beta distribution, α = 1, β = 2.

The third iteration must generate a value between 0 and (100-(W4+W3)), according to the simple Beta function $\alpha = 1$, $\beta = 1$. (See Figure A3 below). We will refer to this value as W2. The fourth distribution is defined as W1 = 100 – (W4+W3+W2).

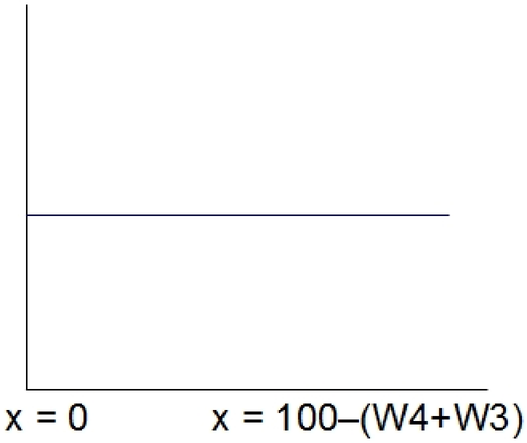

Figure A3. Beta distribution, α = 1, β = 1

To show that the resulting probability distributions of all four criteria uniformly sample the weight space, we examine the frequency diagrams for each weight resulting from a sample run with 32,000 Monte Carlo trials below, starting with W4. The results match the expected Beta distribution where $\alpha = 1$, $\beta = 3$ (Figure A4).

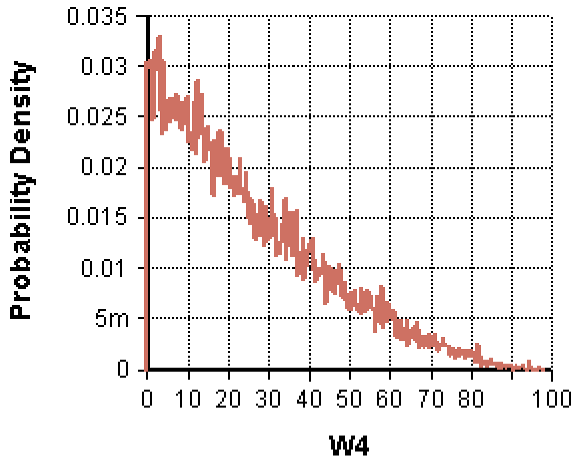

Figure A4. W4 frequency distribution for 32,000 Monte Carlo trials in *Analytica.*

W3 is selected according to a Beta function whose β characteristic is one less, and is constrained such that the maximum possible value is 100 minus the weight sampled for W4. *Notice that the resulting frequency diagram for W3* (Figure A5) *matches that of W4* (Figure A4).

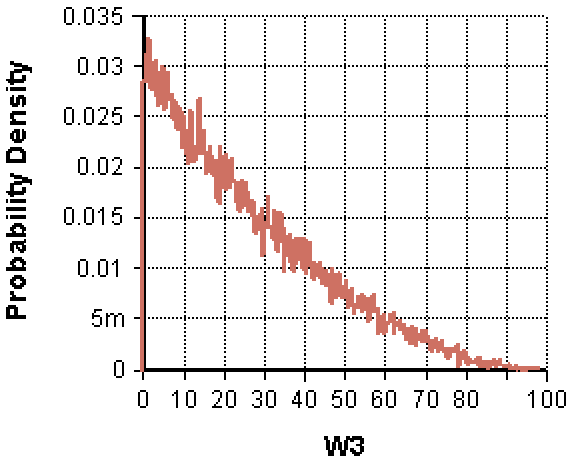

Figure A5. W3 frequency distribution matching that of W4.

Similarly, W2 is selected with a smaller β, and constrained such that the maximum possible value is 100 minus the sum of W3 and W4 (Figure A6). Finally, the fourth variable, which is selected with no random variability, continues to show the proper weight distribution (Figure A7). In each case, the diagrams showing the probability distribution are, for all practical purposes, identical.

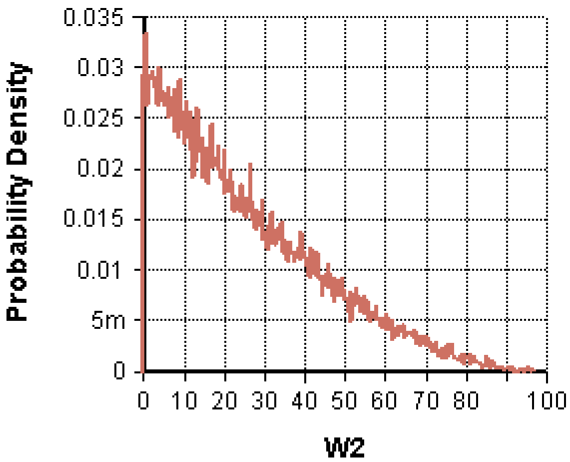

Figure A6. W2 frequency distribution matching that of W4.

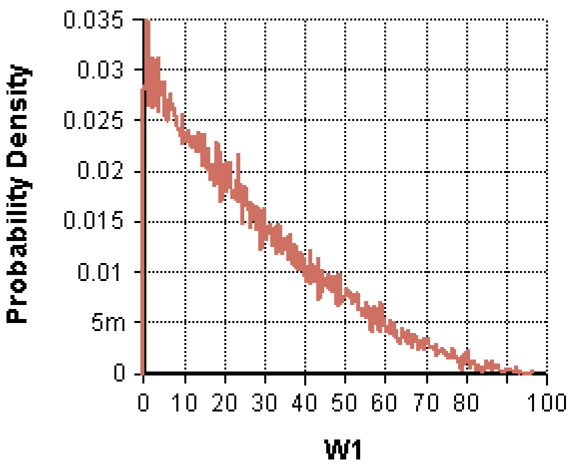

Figure A7. W1 frequency distribution matching that of W4.

An interesting quality of the Beta distribution is that the arithmetic mean is equal to $\frac{\alpha}{\alpha+\beta}$. In our application, α is always 1, and β is always the number of criteria *N*-1. Therefore the mean of each Beta distribution is $\frac{1}{N}$. However, the distribution around that mean is not uniform (as shown in Figures A4-7) and so it is not proper to communicate in terms of a symmetrical weight range such as "25% plus or minus 10%". In this case, a *uniform* distribution of the multi-dimensional weight space means that all mathematical possibilities are explored with equal frequency. However, it remains far more likely that any single criterion weight will be closer to 0 than to 100. Therefore, the upper bound of the distribution will be much farther from the mean than the lower bound.

Establishing Weights with Partial Information: One Constrained Criterion

Having satisfied the unconstrained case representing complete ignorance of stakeholder preferences, we can now turn attention to the more typical case in which information regarding stakeholder preferences can be represented by partial information. For example, a stakeholder may say, "I don't know *exactly* how much weight to assign to fossil fuel savings, but I feel confident it should be at least 50%." In this case, it is necessary to sample all of the possible weights that conform to the stakeholders' preferences.

When the problem involves only three criteria ($N = 3$), it is possible to visualize the feasible weight sets in a three-dimensional diagram. Figure A8 depicts three axes, *x, y,* and *z* that each represent the weights assigned to three decision criteria (respectively). The feasible set weights in the unconstrained case would be represented by the plane with the equation $x + y + z = 100$, intersecting each axes at 100. However, in the special case where $y >= 50$, note that the shape of the feasible weight space remains triangular, albeit smaller. Consequently, the corresponding frequency diagrams with respect to each dimension will maintain the same Beta curve shape ($\alpha = 1$, $\beta = N\text{-}1$), albeit with different minimum and maximum values.

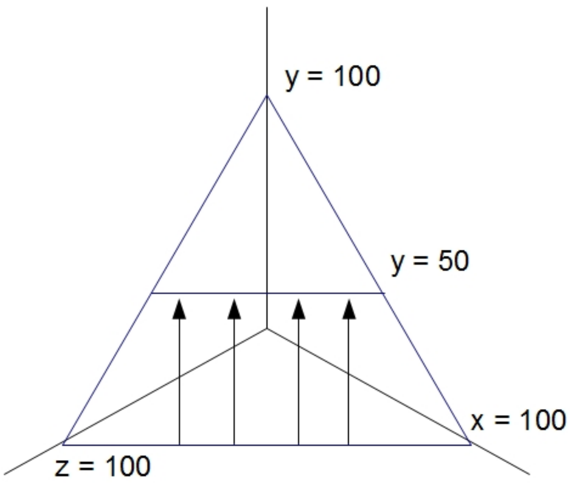

Figure A8. ***x*** **+** ***y*** **+** ***z*** **= 100, with** ***y*** **>= 50.**

For a problem of four dimensions, a set of constraint equations that describe the feasible weights space can be written as:

$W4 + W3 + W2 + W1 = 100$

$W4 \geq W4min$

$W3, W2, W1 \geq 0$

The probabilistic frequency diagram of W4 will now have two components, a random component we call W4' and a constant component called W4min such that W4 = W4' + W4min.

$W4' + W3 + W2 + W1 = (100-W4min)$

$W4', W3, W2, W1 \geq 0$

Figure A9 illustrates the generated probability density of W4 with a W4min of 50.

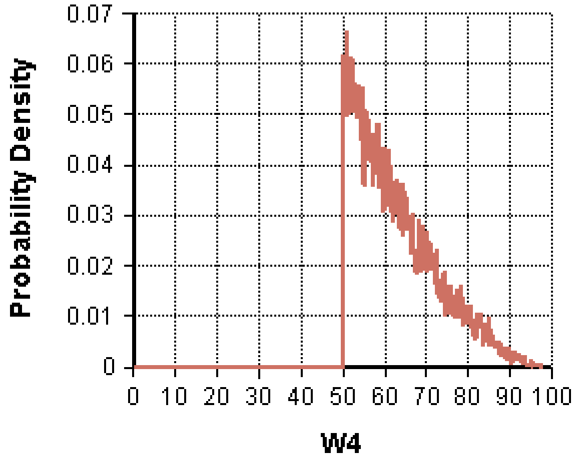

Figure A9. Graph of weight W4 frequency distribution with a minimum constraint W4 >= 50.

The remaining elements of the weight vector maintain the same shape – but only cover values from 0 to 50.

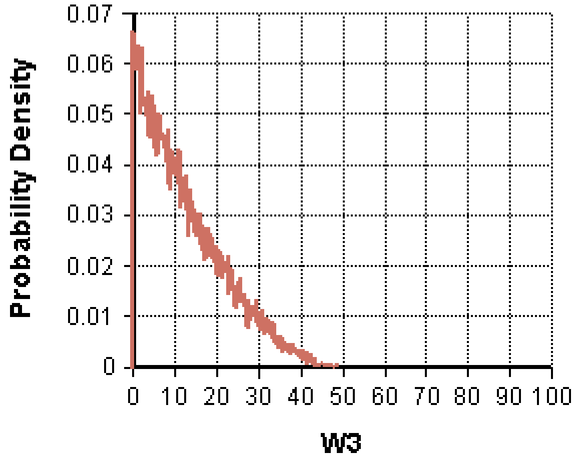

Figure A10. Graph of weight W3 frequency distribution when W4 >= 50.

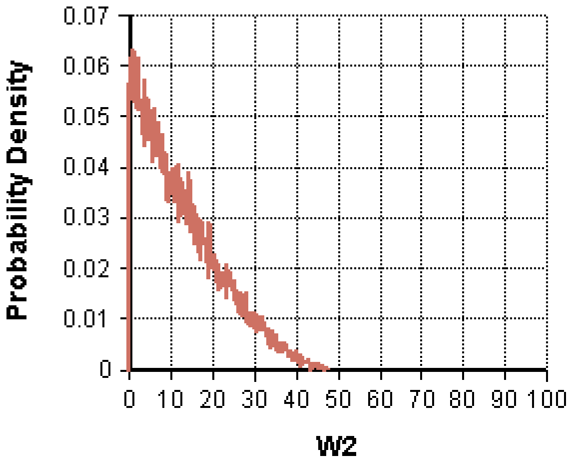

Figure A11. Graph of weight W2 frequency distribution when W4 >= 50

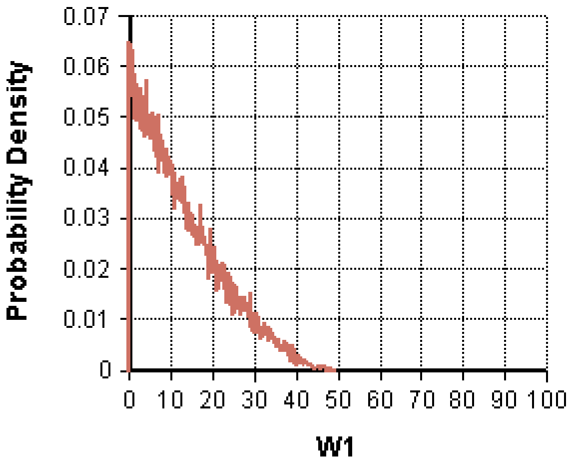

Figure A12. Graph of weight W3 frequency distribution when W4 >= 50.

Multiple Constraints

The case in which one criterion is subject to a minimum constraint may be generalized such that any number of criteria may be subject to a minimum constraint. (Note that this also effectively creates a maximum constraint for each weight as well. A minimum constraint of 10 for W4 creates a maximum constraint for all other weight elements such that none may be greater than 90).

Suppose as a second example that a decision-maker prefers that two criteria weights be subject to different minimum constraints, W4min and W2min.

$W4 + W3 + W2 + W1 = 100$

$W4 \geq W4min$

$W2 \geq W2min$

$W3, W1 \geq 0$

Taking as an example W4min = 20 and W2min = 30, the two minimum constraints create a feasible weights space with a range of 50. Therefore, the maximum for W4 will be 70. The maximum of W2 will be 80. The maximum of W3 and W1 will be 50. All possible combinations within this feasible weights space must be sampled with equal frequency. The resulting frequency distributions are shown in Figures A13, A14, A15 & A16.

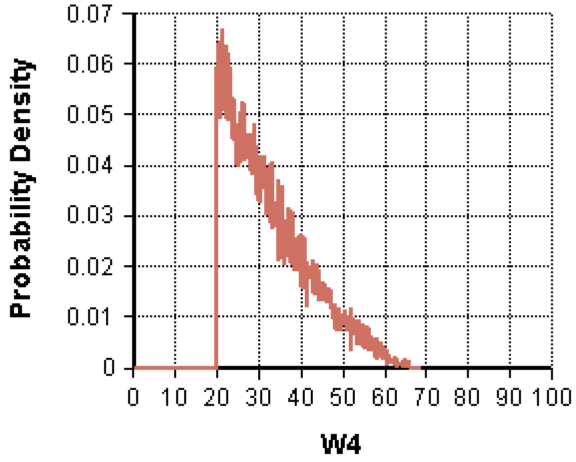

Figure A13. Graph of weight W4 frequency distribution for W4 >= 20, W2 >= 30.

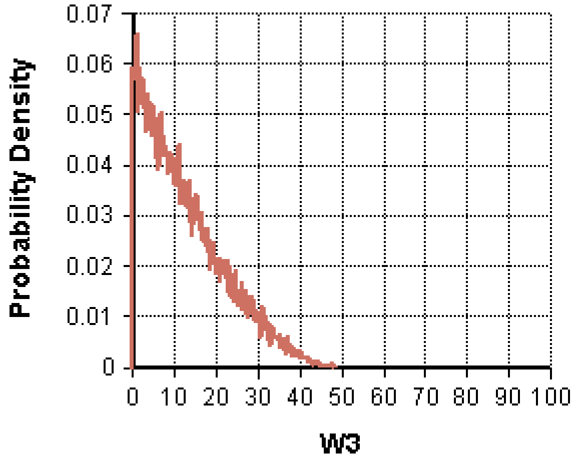

Figure A14. Graph of weight W3 frequency distribution for W4 >= 20, W2 >= 30.

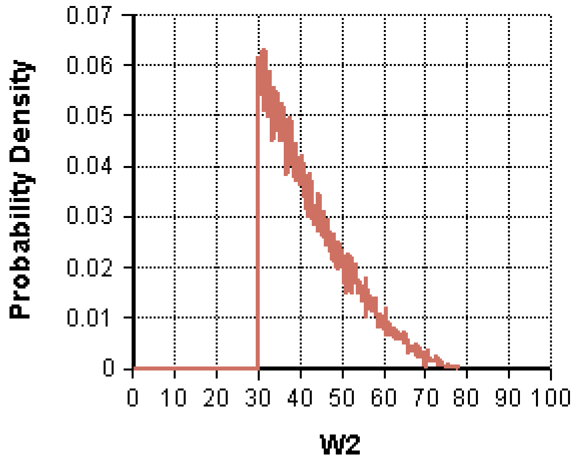

Figure A15. Graph of weight W2 frequency distribution for W4 >= 20, W2 >= 30.

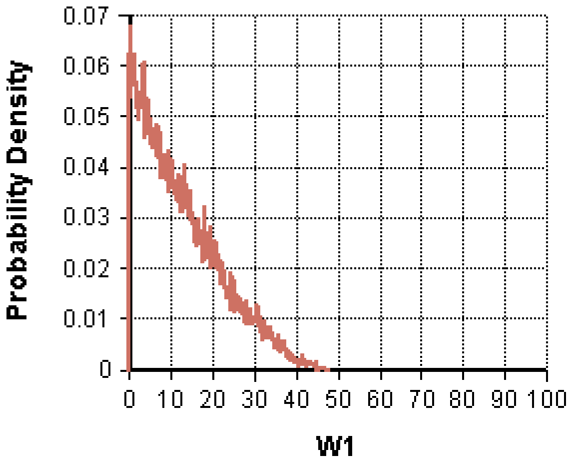

Figure A16. Graph of weight W1 frequency distribution for W4 >= 20, W2 >= 30.

A solution space for any number of criteria with any number of minimum requirements can be created in a similar manner.

Stakeholder weight and confidence elicitation

In SMAA-TY, Stakeholders weigh decision criteria and confidence through simple 5 point scales. These scales have been designed to allow an intuitive answer to the question of "How important is one criterion, relative to others?" without depending upon the stakeholder to be able to address mathematical constraints. Nor are stakeholders asked to detail their weight judgments or rationalize their selection.

The center of the weight scale, Average, maps to the mean weight for the given number of criteria, 100 percent divided by *N*, and truncated for ease of comprehension. Table A1 lists the weights that correspond to each level of the scale, including Well Above Average, Above Average, Average, Below Average, and Well Below Average for problems with 3 to 8 criteria.

	3 Criteria	4 Criteria	5 Criteria	6 Criteria	7 Criteria	8 Criteria
Well Above Average	53%	40%	30%	24%	21%	18%
Above Average	43%	32%	25%	20%	18%	15%
Average	33%	25%	20%	16%	14%	12%
Below Average	23%	18%	15%	12%	10%	9%
Well Below Average	13%	10%	10%	8%	7%	6%

Table A1: Weight specifications based on number of criteria.

In some cases a stakeholder may find it reasonable to declare that all criteria are above or well above average, leaving no criteria below average. To avoid violation of mathematical constraints, SMAA-TY adjusts each weight proportionally such that the sum of all weights adds to 100%. For example, in a four criteria problem, a stakeholder could select one criterion as Well Above Average (40%), two Above Average (32% each), and one at Average (25%). The sum total of each of these is 129%. Consequently, the system will rescale the Well Above Average to 31.0% (which is 40% /

129%), both Above Average criteria to 24.8%, and the Average to 19.4%. A stakeholder with some selections both above and below average will have a less drastic adjustment, and all weights will retain the same relative differentiation as selected. The adjusted weights are not shared with the stakeholder.

Figures A17-19 illustrates the weight distribution for the criteria in the above example and the minimally constraining confidence setting of Fair.

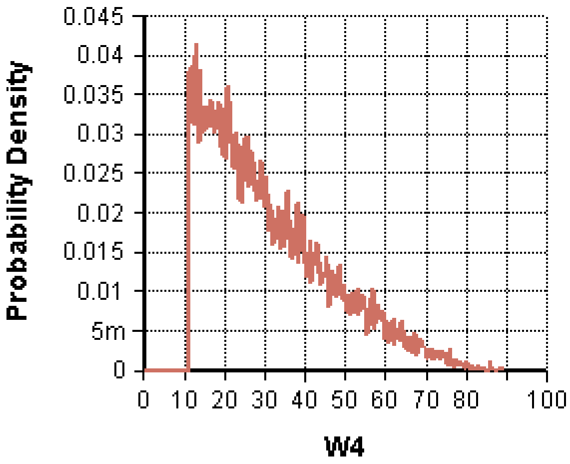

Figure A17. W4 frequency distribution for Well Above Average weight (31%) and Fair confidence.

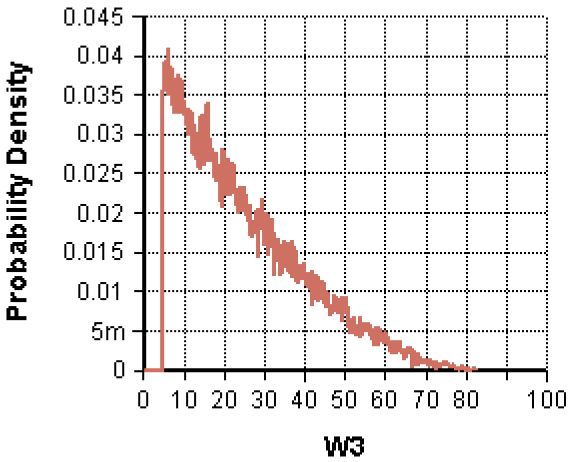

Figure A18. W3 (& W2) frequency distribution for Above Average weight (24.8%) and Fair confidence.

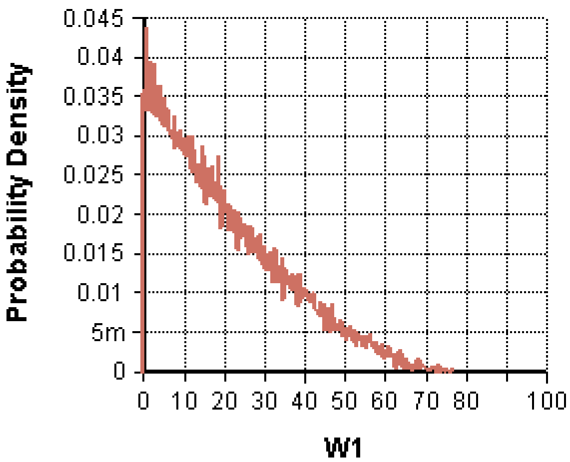

Figure A19. W1 frequency distribution for Average weight (19.4%) and Fair confidence.

In addition to specifying variability in relative weight preferences, SMAA-TY allows stakeholders to choose the uncertainty around the mean weights as Precise, Very Accurate, Accurate, Good, and Fair. The Fair level of confidence corresponds to weight distributions that are largely unconstrained i.e., this constraint allows modest variability in the mean weights through an arbitrary minimum weight level of 5% for Average selections in a 4 criteria problem. All other levels of confidence further narrow the uncertainty.

In a four criteria problem, a stakeholder with Fair confidence results in a range of uncertainty with a lower bound set 20% below the adjusted weight (of each criterion, respectively). As a result of these lower bounds, the mathematical constraint that dictates all weights must equal 100% results in an upper bound on the range that is 60% (i.e, the 20% below lower bound multiplied by N-1) above the adjusted weight. If the stakeholder selected Average weights for all criteria, the resulting adjusted weight would be 25%, with a lower bound of 5% and upper bound of 85%. While establishing the lower bound at 5% is arbitrary, it represents the reasonable assumption that stakeholders would not include a criterion in the analysis unless that criterion represented some minimal level of concern. Table A2 shows the set-point of the lower bound relative to the adjusted weight, for different levels of confidence in problems between 3 and 8 total criteria. For simplicity, SMAA-TY applies the same confidence interval to all criteria. The confidence percentages are not shared with the stakeholder.

It is possible under conditions of low confidence and Below Average or Well below Average weighting that the lower bound of a criterion weight range suggested in Table A2 could fall below zero. In this case, 0% is assigned as the lower bound of the range.

	3 Criteria	4 Criteria	5 Criteria	6 Criteria	7 Criteria	8 Criteria
Precise	-3%	-2%	-2%	-1%	-1%	-1%
Very Accurate	-5%	-5%	-4%	-3%	-3%	-2%
Accurate	-10%	-10%	-8%	-7%	-6%	-5%
Good	-15%	-15%	-12%	-10%	-8%	-7%
Fair	-22%	-20%	-16%	-13%	-11%	-10%

Table A2: Lower bound of weight range, reported relative to adjusted weight, for all levels of confidence and in problems containing 3 to 8 criteria. The upper bound for any one criterion is the sum of the lower bounds bound for all other criteria, subtracted from 100.

Figures A20-24 illustrate frequency distributions that result from the previous example in which criterion W4 is weighted Well Above Average, two other criteria are weighted as Above Average, and one is merely Average. For comparison, Figure A17 showing the broader W4 frequency distribution calculated under Fair confidence (that is indicative of greater uncertainty) has been reprinted as A20 with a scale common to figures A21-A24. Figure A21 shows the frequency distribution for W4 under Good confidence and each subsequent figure portrays increasing confidence from Accurate (Figure A22), to Very Accurate (Figure A23) and Precise (Figure A24). In each case the mean weight specified by the stakeholder remains unchanged.

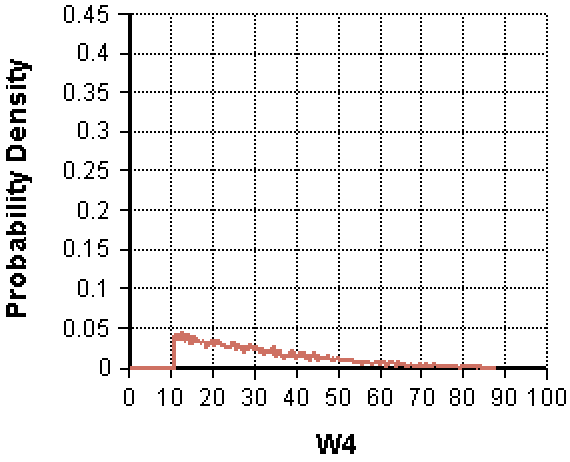

Figure A20. W4 frequency distribution for Well Above Average Weight (31%) and Fair confidence (re-scaled).

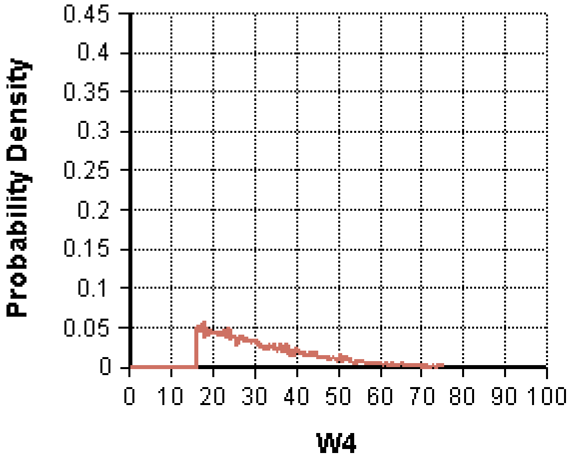

Figure A21. W4 frequency distribution for Well Above Average Weight (31%) and Good confidence.

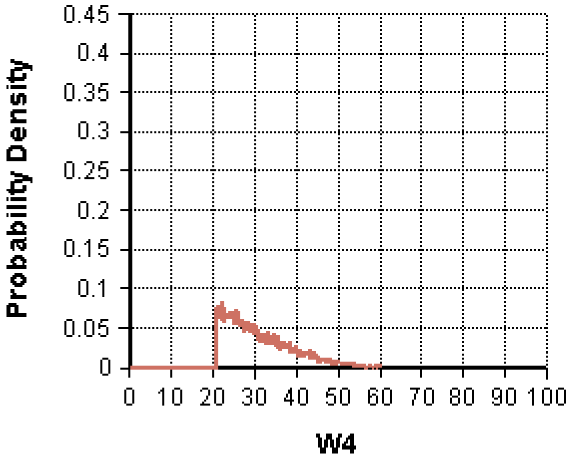

Figure A22. W4 frequency distribution for Well Above Average Weight (31%) and Accurate confidence.

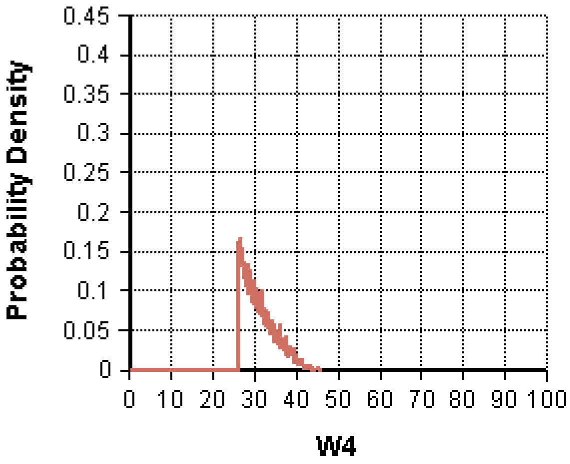

Figure A23. W4 frequency distribution for Well Above Average Weight (31%) and Very Accurate confidence.

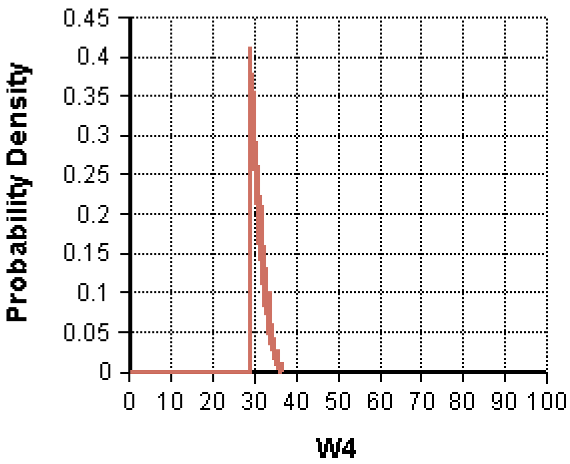

Figure A24. W4 frequency distribution for Well Above Average Weight (31%) and Precise confidence.